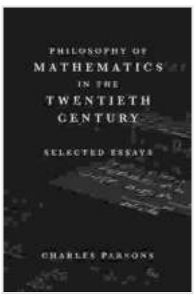
Philosophy Of Mathematics In The Twentieth Century: Unraveling the Enigma

The 20th century witnessed a remarkable surge of philosophical inquiries into the very nature of mathematics. This intellectual endeavor, known as the philosophy of mathematics, sought to illuminate the foundations, methods, and implications of mathematical knowledge.



Philosophy of Mathematics in the Twentieth Century: Selected Essays by Charles Parsons

5 out of 5

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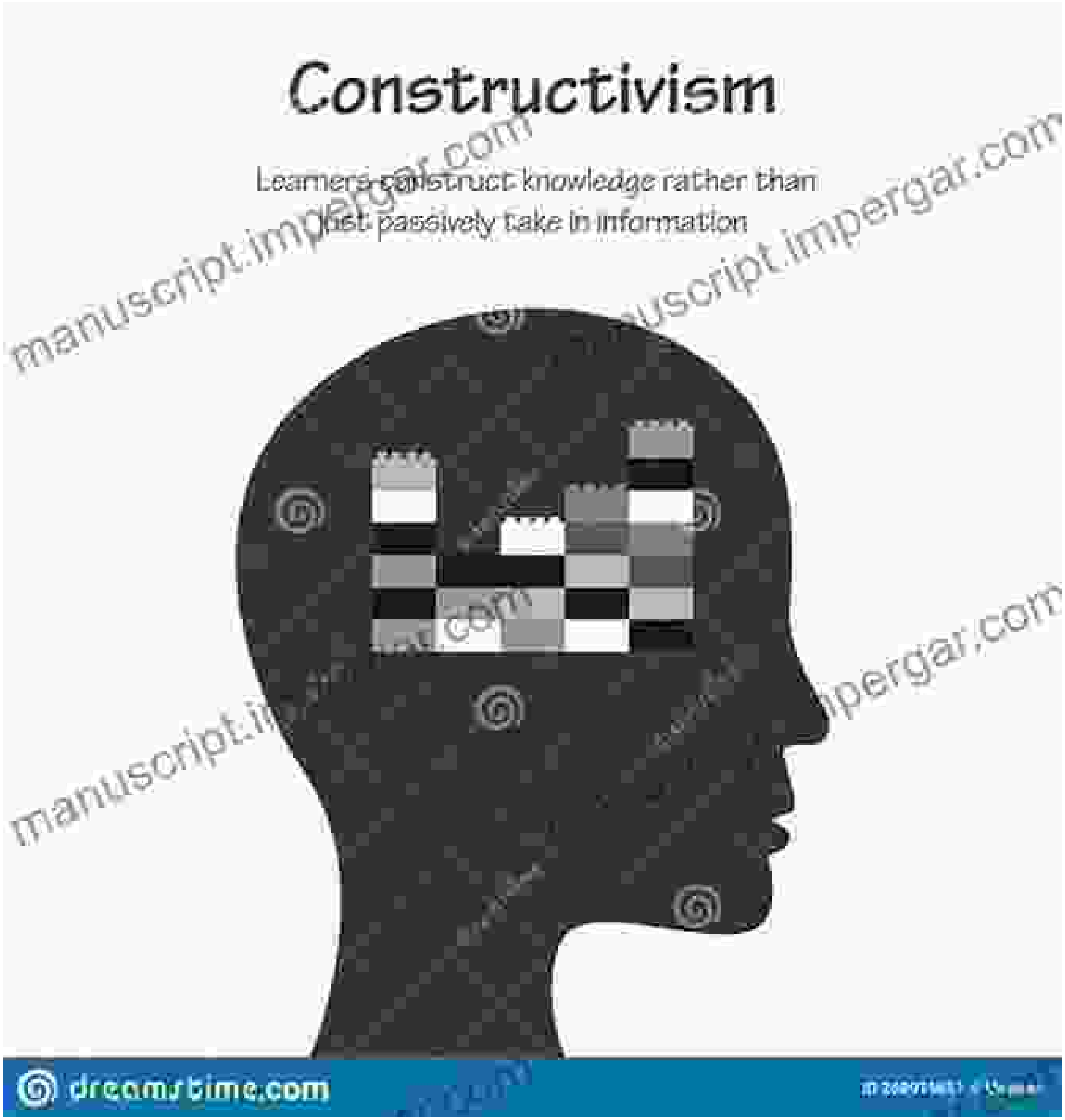
Logical Positivism and the Verification Principle

Logical positivism, a dominant philosophical movement of the early 20th century, exerted a significant influence on the philosophy of mathematics. Logical positivists, such as Moritz Schlick and Rudolf Carnap, argued that meaningful statements must be verifiable through empirical observation or logical reasoning.

In the context of mathematics, this meant that mathematical statements, such as "2 + 2 = 4," were considered true solely based on their logical consistency within the axiomatic system of mathematics. Metaphysical claims about the objective existence of mathematical objects, on the other hand, were deemed meaningless because they could not be empirically tested.

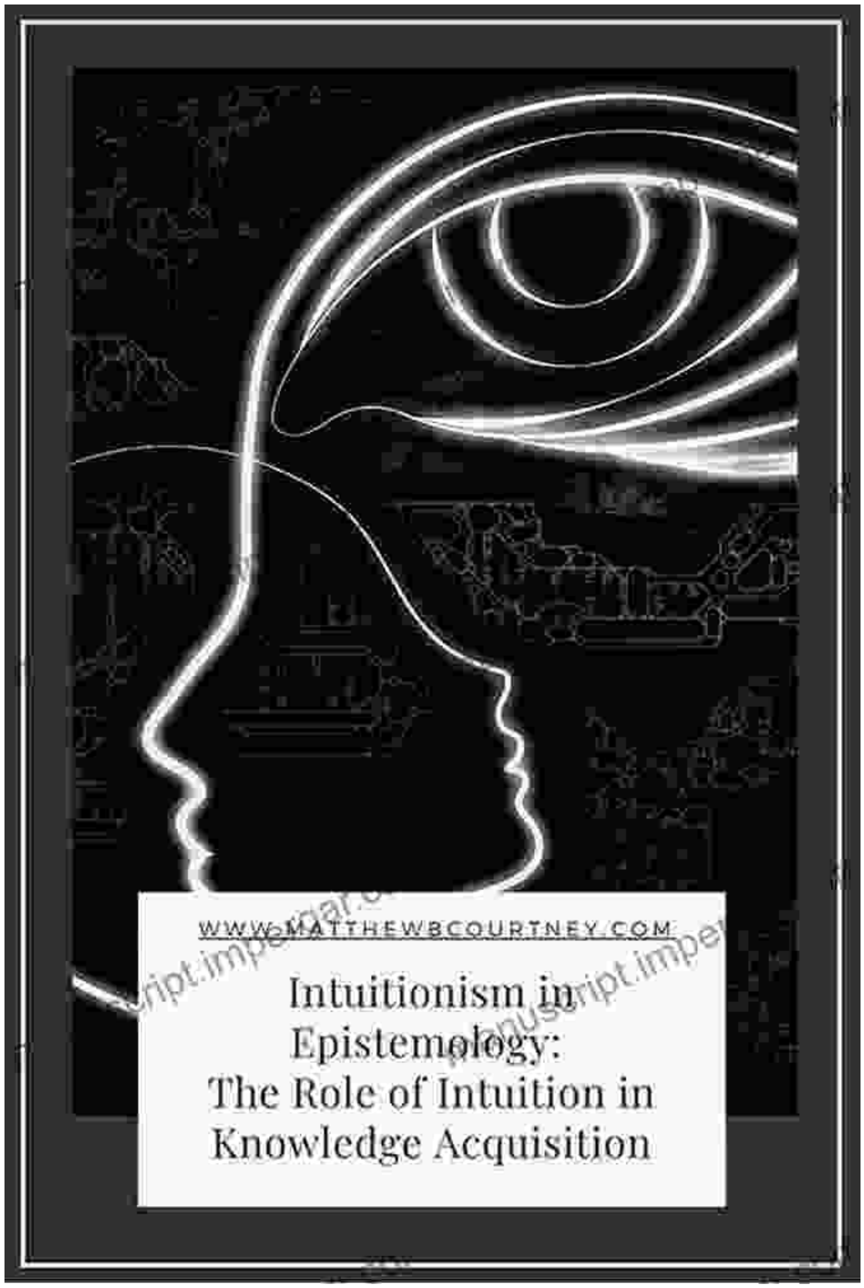
Constructivism and the Rejection of Platonism



Constructivism emerged as an alternative perspective to logical positivism's verification principle. Constructivists, like L.E.J. Brouwer and Errett Bishop, argued that mathematics is not a static collection of truths but rather a dynamic process of human construction.

They rejected the Platonic notion of mathematical entities existing in an independent, abstract realm. Instead, they maintained that mathematical concepts and objects are products of human imagination and are subject to ongoing revision and refinement.

Intuitionism and the Role of Intuition

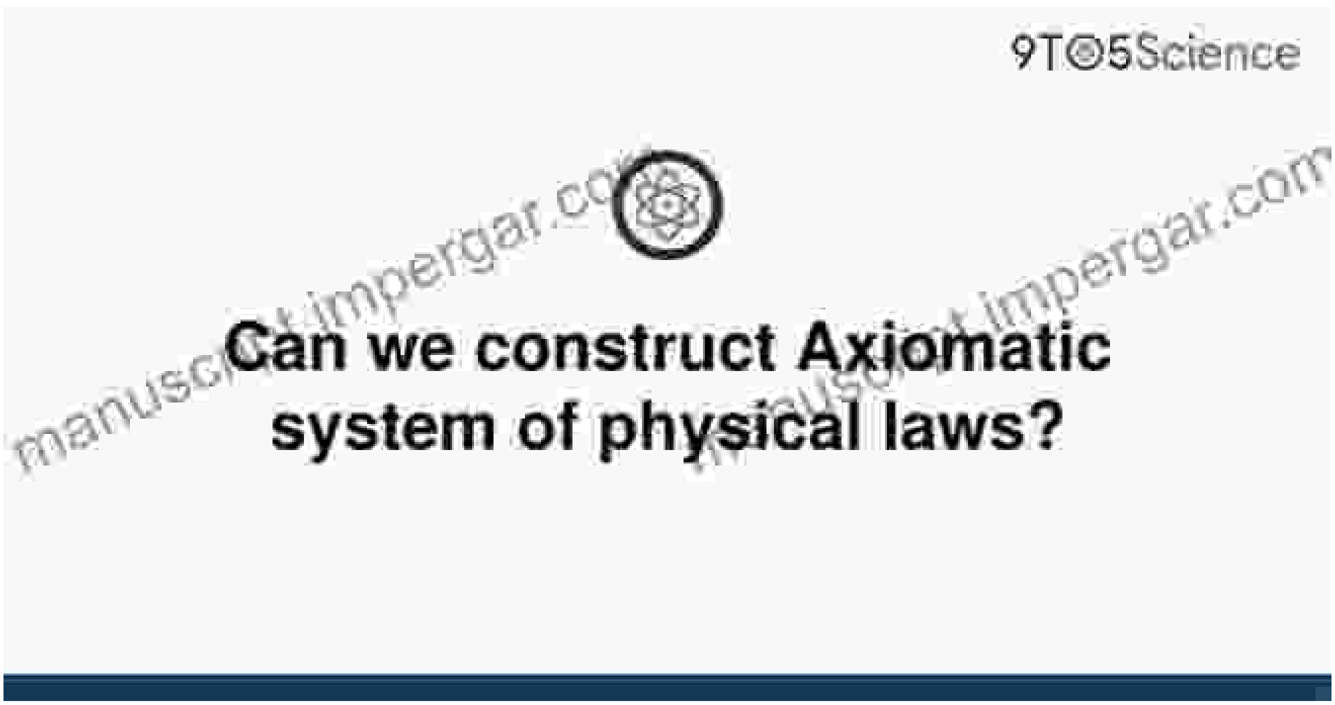


Intuitionism, a philosophical movement closely aligned with constructivism, further challenged the emphasis on formal logic in mathematics.

Intuitionists, such as Henri Poincaré and Arend Heyting, contended that intuition plays a fundamental role in mathematical reasoning.

They argued that certain mathematical principles, such as the law of excluded middle, which states that every proposition is either true or false, cannot be proven through purely logical means and must be accepted intuitively.

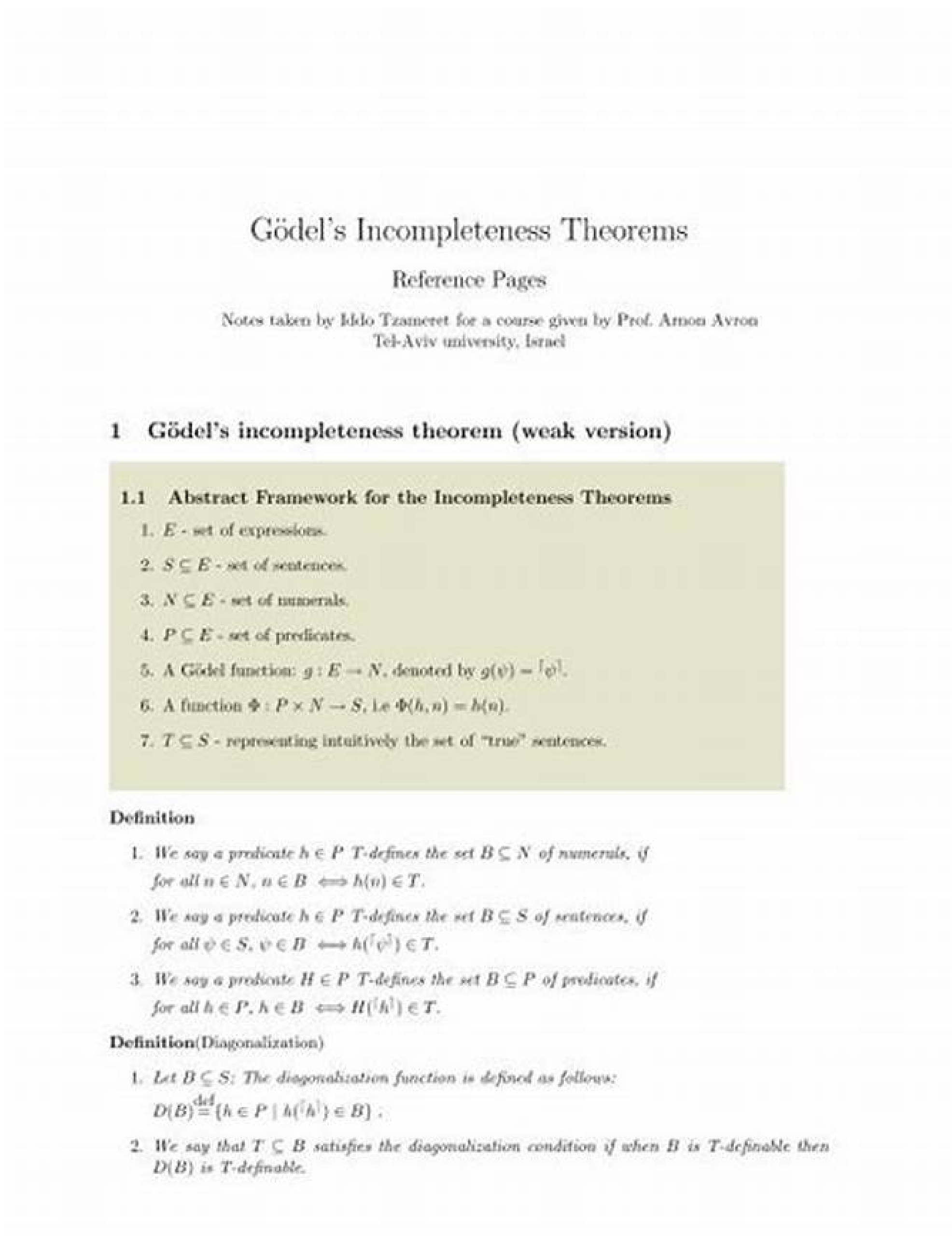
Formalism and the Search for an Axiomatic Foundation



Formalism, in contrast to constructivism and intuitionism, sought to establish a secure foundation for mathematics through axiomatization. Formalists, led by David Hilbert and Kurt Gödel, aimed to develop a complete and consistent set of axioms that would serve as the irrefutable basis for all mathematical knowledge.

They envisioned mathematics as a purely formal system, akin to a game played according to predefined rules, divorced from any real-world applications or intuitive meanings.

Gödel's Incompleteness Theorems: A Foundational Crisis



Gödel's Incompleteness Theorems

Reference Pages

Notes taken by Iddo Tzameret for a course given by Prof. Arnon Avron
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1 Gödel's incompleteness theorem (weak version)

1.1 Abstract Framework for the Incompleteness Theorems

1. E - set of expressions.
2. $S \subseteq E$ - set of sentences.
3. $N \subseteq E$ - set of numerals.
4. $P \subseteq E$ - set of predicates.
5. A Gödel function: $g : E \to N$, denoted by $g(\psi) = \ulcorner \psi \urcorner$.
6. A function $\Phi : P \times N \to S$, i.e $\Phi(h, n) = h(n)$.
7. $T \subseteq S$ - representing intuitively the set of "true" sentences.

Definition

1. *We say a predicate $h \in P$ T-defines the set $B \subseteq N$ of numerals, if for all $n \in N$, $n \in B \iff h(n) \in T$.*
2. *We say a predicate $h \in P$ T-defines the set $B \subseteq S$ of sentences, if for all $\psi \in S$, $\psi \in B \iff h(\ulcorner \psi \urcorner) \in T$.*
3. *We say a predicate $H \in P$ T-defines the set $B \subseteq P$ of predicates, if for all $h \in P$, $h \in B \iff H(\ulcorner h \urcorner) \in T$.*

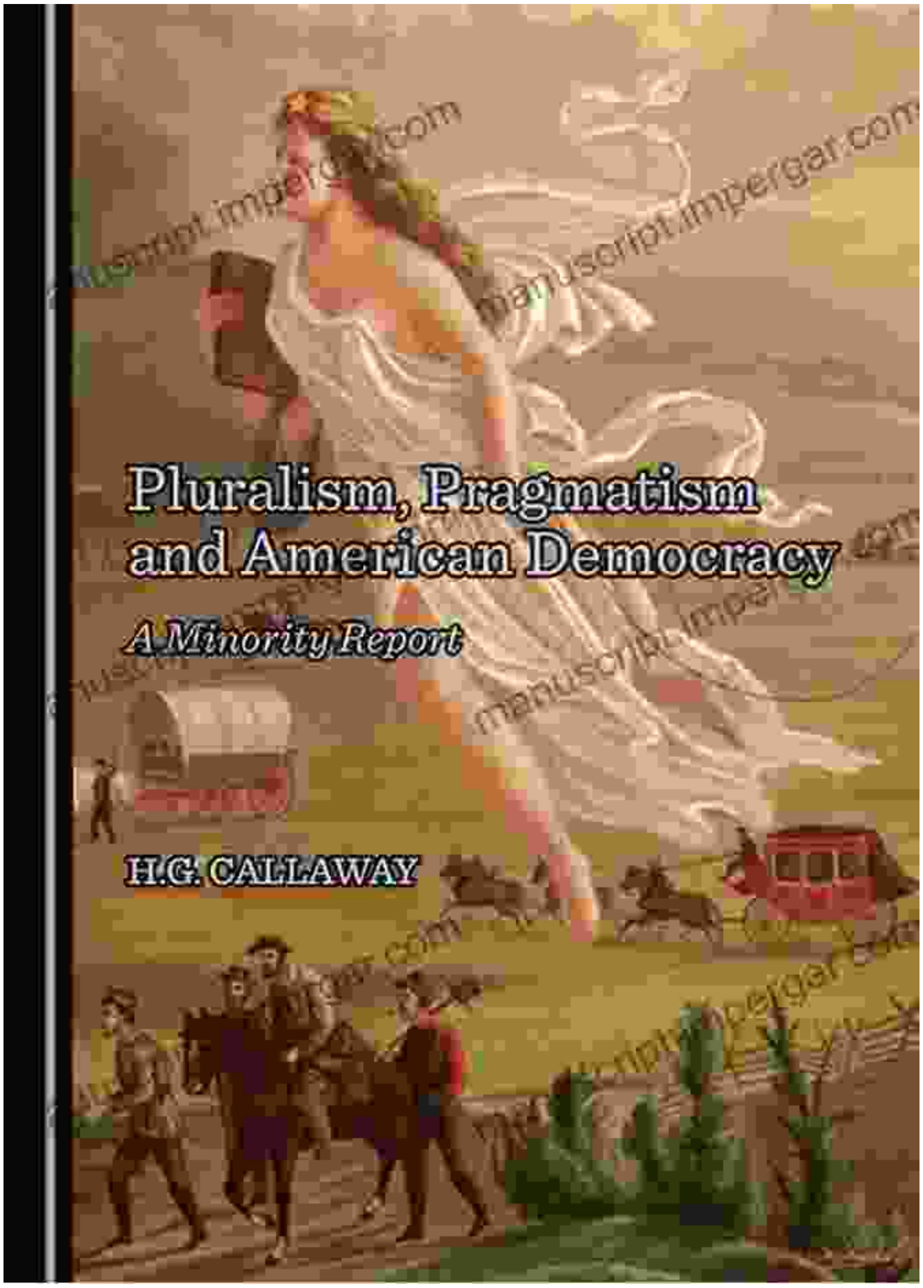
Definition(Diagonalization)

1. *Let $B \subseteq S$: The diagonalization function is defined as follows:* $D(B) \overset{\text{def}}{=} \{h \in P \mid h(\ulcorner h \urcorner) \in B\}$.
2. *We say that $T \subseteq B$ satisfies the diagonalization condition if when B is T-definable then $D(B)$ is T-definable.*

Gödel's groundbreaking incompleteness theorems dealt a decisive blow to the formalist program. These theorems demonstrated that any consistent axiomatic system capable of expressing basic arithmetic is either incomplete, meaning that there are true statements that cannot be proven within the system, or inconsistent, meaning that it contains contradictory statements.

This shattered the formalist aspiration of constructing a comprehensive and self-contained foundation for mathematics, forever leaving open the possibility of undecidable statements and the potential for foundational uncertainty.

Late 20th Century Developments: Pluralism and Pragmatism



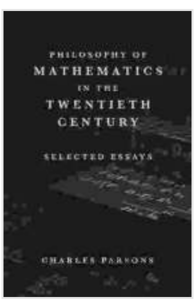
The late 20th century witnessed a shift towards pluralism and pragmatism in the philosophy of mathematics. Philosophers began to acknowledge the diversity of mathematical practices and the importance of practical considerations in shaping mathematical knowledge.

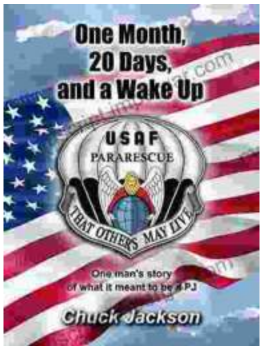
They moved away from the grand foundational debates of earlier decades and instead focused on exploring the various ways in which mathematics is used and understood in different contexts, from scientific modeling to computer science to everyday life.

: Unveiling the Multifaceted Nature of Mathematics

The philosophy of mathematics in the 20th century was a vibrant and multifaceted endeavor that challenged traditional assumptions about the nature of mathematical knowledge. It gave rise to diverse and often conflicting schools of thought, each offering unique insights into the intricate relationship between mathematics, logic, and human experience.

Through this philosophical journey, we have unveiled the enigmatic nature of mathematics, recognizing it as a dynamic and evolving human endeavor that transcends narrow definitions and embraces the diversity of human thought and practice.





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